2.71/2.710 Optics HW5 **Spring '14** Posted March 31, 2014, Due April 9, 2014

- 1. (Adapted from Voelz 4-4) Derive analytic expressions for the Fraunhofer field and irradiance patterns for the following apertures (shown in Fig. 4.5) illuminated with a unit amplitude plane wave. Plot the analytic Fraunhofer irradiance pattern images and profiles for the above apertures on the computer. Choose suitable propagation distances z and side lengths L in the observation plane. Use $\lambda = 0.547 \mu m$ and the following parameters:
 - (a) $w_X = 0.1 \text{ mm}, w_Y = 0.05 \text{ mm}.$
 - (b) $w_1 = 1 \text{ mm}, w_2 = 0.2 \text{ mm}.$
 - (c) w = 0.3 mm, $\Delta s = 1 \text{ mm}$.



© SPIE Press. All rights reserved. This content is excluded from our Creative Commons license. For more information, see http://ocw.mit.edu/fairuse.

Voelz Figure 4.5 Apertures: (a)rectangle; (b)circles with obscuration; (c) pair of circles.

- 2. (Adapted from Pedrotti 11-14) Retina Display is a brand name used by Apple which they claim have a high enough pixel density that the human eye is unable to notice pixelation at a typical viewing distance. Let's find the required pixel size based on principle of diffraction. Assume that the pupil diameter of a normal eye typically can vary 2 to 7 mm in response to ambient light variation.
 - a. What is the corresponding range of distance over which such an eye can detect the separation of objects 1 mm apart.
 - b. Experiment to find the range of distances over which you can detect the separation of lines placed 1mm apart. Use the results of your experiments to estimate the diameter range of your pupils.
 - c. Now we assume the screen of i-Phone is placed at 25cm away from the eye (you may recall this is so called the nearpoint of the eye), estimate the maximum size of pixels on the screen you would not notice the pixilation effect.
- **3.** Newton's Rings: The following figure illustrates a setup used for testing plano-convex lenses using a famous interference effect, known as Newton's rings. A plano-convex lens (n=1.5, thickness t= 1cm, and radius of curvature R=20cm) is placed on a flat mirror, and illuminated at normal incidence with a plane monochromatic light at wavelength λ =500nm. For the approximation, assume the path difference between the two reflected beams (one at the curved surface of the plano-convex lens and the other at the mirror surface) is much smaller than the radius of curvature R of the lens, so we are looking at the inner zones of Netwon's rings.





- a) Assume both the plano-convex lens (n=1.5) and the beam splitter are large enough to accommodate the entire lateral width of the expanding paraxial wave, write an analytical expression for and sketch the interference pattern as function of the coordinate *x* at the observation plane (CCD detector).
- b) Calculate the radius of 10th bright ring on the CCD detector.
- c) Explain how this Newton's ring pattern would change if the illumination beam tilted by a small angle θ .
- **4.** A **blazed glass grating** (index of refraction n=1.5) displays a surface shaped like a sawtooth, as shown in the following figure. The pitch of the tooth is 2.5 μm.



- a) Find the optimum height *h* that maximizes the peak intensity of the -1 order diffracted light for incident wavelength λ = 532nm.
- b) Using the grating height from a), calculate the steering angle of the -1 order diffracted beam for the following wavelengths: λ= 488nm (Blue), 532nm (Green), and 633nm (Red).

5. Diffractive lens (Adpated from Goodman 5-13, 2.710 only): A diffracting structure has a symmetric amplitude transmittance function given by

$$t(r) = \left[\frac{1}{2} + \frac{1}{2}\cos(\gamma r^2)\right] \operatorname{circ}\left(\frac{r}{R}\right)$$

The grating is placed at the plane z = 0 and illuminated by an on-axis plane wave $E(x, z = 0) = E_0$

- **a)** Show that this grating also possess focusing function.
- **b)** Determine the focal length of this grating(Hint: you may compare the transmittance function with that of a thin lens $t(r) = exp\left[-ik\frac{r^2}{2f}\right]circ(\frac{r}{R})$)

c) What are the limitations of using this diffractive grating as an imaging element for polychromatic objects?

2.71 / 2.710 Optics Spring 2014

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.